A COHESIVE ZONE MODEL FOR THE INVESTIGATION OF THE BREATHING MECHANISM OF TRANSVERSAL CRACKS IN ROTORS

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Abstract. The presence of a crack reduces the mean stiffness of the rotor system and introduces a stiffness variation during the revolution of the shaft. How the variable part of the rotor stiffness varies between a minimum (for a closed crack) and a maximum (for an open crack), depends on the so-called breathing mechanism. The breathing mechanism is known when the open and closed parts of the cracked area are known for all angular positions of the rotor. Here, finite element (FE) and multi-body simulation (MBS) is introduced. It is based on a representation of the fracture process zone by a cohesive zone model (CZM). First, the cracked elastic shaft with various relative crack depths is modelled by FE. As a second step, the FE model of the shaft is transferred into an MBS model in order to analyze the dynamic loads, due to the crack, and the inertia force acting during rotation at different rotating speeds. Finally, the vibration responses in the centroid of the shaft obtained from the MBS have been exported into the FE model in order to observe the breathing mechanism. This proposed technique provides a useful tool for the analysis of rotor systems containing cracks, reveals the shape of the open crack part during rotation and helps investigating the dynamic behaviour of cracked shafts.

Keywords: Cohesive zone model, Breathing mechanism, Stiffness variation, Finite element, Multi-body simulation.

1. INTRODUCTION

Fatigue cracking of rotor shafts has long been identified as a limiting factor for safe and reliable operation of turbomachines. It can lead to catastrophic failure and great economic loss if not detected early. A crack in the rotor causes local changes in stiffness. These changes, in turn, affect the dynamics of the system: frequency of the natural vibrations and the amplitudes of forced vibrations are changed. If a cracked shaft rotates under external loading, the crack opens and closes regularly during the revolution of the shaft, it breathes. The breathing mechanism is produced by the stress distribution around the crack mainly due to the action of bending moment, while the effect of torsion is negligible. Usually, shaft cracks breathe when crack sizes are small, running speeds are low and radial forces are large [1]. Breathing of the
crack comes from the fact that the static deflection of the shaft is much greater than the deflection due to the dynamic response of the cracked rotor. The influence of a breathing crack on the vibration of a rotating shaft has been in the focus of many researchers. Comprehensive literature surveys of various crack modelling techniques, system behaviour of cracked rotor and detection procedures to diagnose fracture damage were contributed by Sabnavis et al. [2] and Kumar and Rastogi [3]. More recent studies have been reviewed by Bachschmid et al. [4]. They noted that the breathing mechanism of cracks in rotating shafts can be accurately investigated by means of 3D non-linear finite element models. Based on these simulations, a simple approximation of the breathing mechanism, describing the location and extension of the crack closure line during rotation, can be established. The breathing mechanism is the result of the stress and strain distribution around the cracked area, which is due to: static loads, like the weight, bearing reaction forces and dynamical loads, due to unbalance and vibration induced inertia force distribution [5]. The accurate modelling of the breathing behaviour, that is the gradually opening and closing of the crack during one rotation of the shaft, still needs further investigations. An original method for calculating the constitutive law of a cracked beam section under bending has been proposed by Andrieux and Varé [6]. Based on three-dimensional computations taking into account the unilateral contact between the lips of the crack, it consists in defining a constitutive relation between the bending moment applied to the cracked section and the resulting field of displacements, compatible with the beam theory so that it can be used in rotor-dynamics software. Varé and Andrieux [7] extended this method in order to show how shear effects can be implemented in the model of a cracked section. Arem und Maitournam [8] presented stiffness variations deduced from three-dimensional FE calculations accounting for the unilateral contact between the crack lips as originally developed by Andrieux and Varé [6].

The strain energy release rate approach combined with stress intensity factors allows the additional flexibility introduced by the crack to be calculated when the crack is open. However, this approach is not valid when the crack is partially open due to the breathing mechanism. The extension of this approach to the breathing crack is affected by some errors due to the fact that the crack tip is supposed to be formed by the boundary between the cracked areas and the uncracked areas at the time of closing crack, because on this boundary no stress intensity factor will appear. Moreover, the cracked cross section is not anymore planar, but is distorted. This is not taken into account by the stress intensity factor approach [5]. A realistic model of a breathing crack is a difficult issue to the lack of fundamental understanding about certain aspects of the breathing mechanism. This involves not only the identification of variables affecting the breathing crack behaviour, but also the issues for evaluating the structural dynamic response of the fractured material. It is also not yet entirely clear how partial closure interacts with the key variables of the problem [9].

The cohesive zone model (CZM) has been widely used as an alternative to stress intensity factor based fracture mechanics. It can deal with the nonlinear zone ahead of the crack tip due to plasticity or microcracking. The CZM describes material failure on a more phenomenological basis (i.e. without considering the material microstructure). The general advantage of this model when compared to classical fracture mechanics is that the parameters of the respective models depend only on the material and not on the geometry. This concept
guarantees transferability from specimen to structure over a wide range of geometries. The origin of the cohesive zone concept can be traced back to the strip yield model proposed by Dugdale [10] and Barenblatt [11] in which the narrow zone of localized deformation ahead of the crack tip was substituted by cohesive traction between the bounding surfaces. The constitutive behaviour which causes the cohesive elements to open and eventually to fail is described by the so called traction-separation law. It relates the traction vector to the displacement jump across the interface and is usually called separation. The energy dissipated by the element until total failure is derived as the integral of the traction-separation curve. However, the traction-separation law depends on the stress state, which can be characterised by the tri-axiality, which is the hydrostatic stress divided by the von Mises equivalent stress. This issue was first investigated by Siegmund and Brocks [12], [13]. The approach was extended to simulation of dynamic ductile crack growth by Anvari et al. [14] and Scheider [15]. Banerjee and Manivasagam [16] proposed a versatile CZM to predict ductile fracture at different states of stress. The formulation developed for mode-I plane strain accounts explicitly for triaxiality of the stress-state by using basic elastic-plastic constitutive relations combined with two new model parameters, which are independent of the stress-state.

This article introduces a simulation method based on a finite element (FE) model and multi-body simulation (MBS) in order to study the breathing of an elastic shaft with a non-propagating edge crack. The CZM formulation is implemented in an FE model. A cracked elastic shaft with various relative crack depths is modelled by FE, then transferred into an MBS model in order to analyze the dynamic loads acting during rotation at different rotating speeds. The proposed method is assessed for the case of a cracked shaft loaded by weight only. The vibration responses in the centroid of the shaft obtained from MBS have been exported into FE to observe the breathing mechanism. The accuracy of the results is demonstrated through comparisons with the results available in the literature.

2. FINITE ELEMENT MODELLING WITH COHESIVE ZONE MODEL

2.1. Basis of the cohesive process zone model

The fracture process zone model is shown in Figure 1, where the fracture process zone can be defined as the region within the separating surfaces where the surface traction values are nonzero. Processes occurring within the process zone are accounted for through traction-displacement relations. In classical fracture mechanics, the crack growth problem is identified as a moving boundary value problem in which the primary unknown is usually the trajectory of a single point referred to as crack tip. CZM represents a zone or a region where material separates, the location of a crack tip within the fracture process zone cannot be uniquely identified. Formation and extension of this surface require that the maximum principal stress reaches a given value, namely the cohesive strength of the material. When this occurs, the crack surface initiates or grows perpendicularly to the direction of the maximum principal stress. The two faces of the surface exert on each other equal and opposite tensile stresses (cohesive stresses) whose values are a unique function $f(\delta)$ of the separation $\delta$ between the faces. When the separation reaches another given value (the critical separation, $\delta_c$), the cohesive stress becomes null and fracture takes place.
The fracture behaviour of each material is described by the cohesive traction as the function

$$\sigma_n = \sigma_{\text{max}} f(\delta)$$

(1)

where $\sigma_{\text{max}}$ is the peak value of traction. The function $f(\delta)$ defines the shape of the traction-separation law and the area under the cohesive law curve is the work of separation or cohesive energy $G_c$

$$G_c = \int_0^{\delta_c} \sigma_n(\delta) \, d\delta.$$  

(2)

Since the CZM is a phenomenological model, various formulations for defining the shape of traction-separation law and the cohesive values are in use [17]. A versatile CZM to predict ductile fracture at different states of stress is proposed in [16]. The formulation developed for mode-I plane strain accounts explicitly for triaxiality of the stress-state by using basic elastic-plastic constitutive relations combined with two stress-state independent new model parameters. The proposed traction-separation law has three distinct regions of constitutive behaviour: the traction separation law is linear up to the separation limit $\delta_n = \delta_1$ exhibits strain hardening up to $\delta_n = \delta_2$ followed by a softening curve. The relevant variables and zones are sketched in Figure 2.
2.2. Cohesive zone model formulation

When fracture proceeds, energy must be supplied by external loads. The bounding material undergoes elasto-plastic deformation involving elastic energy and plastic dissipative energy. In addition to plasticity, energy is supplied to the fracture process zone in form of cohesive energy that is dissipated within the cohesive elements. The cohesive energy is the sum of the surface energy and all dissipative processes that take place within the crack tip regime. For the present problem, a perfect energy balance between external work $W$ and the sum of elastic energy $E_{el}$ and cohesive energy $G_c$ [17], [18] will be assumed. The energy balance is given by

$$W = E_{el} + G_c$$

$$W = \int_0^t \left( \int_V \sigma : \dot{\varepsilon}_{el} \, dV \right) \, dt + \int_0^t \left( \int_S \sigma_n : \dot{\delta} \, dS \right) \, dt$$

where $\sigma$, $\dot{\varepsilon}_{el}$, $\sigma_n$, and $\dot{\delta}$ are nominal stress tensor, elastic strain rate, cohesive traction and cohesive separation rate, respectively (including the terms for specimen volume $V$ and the internal specimen surface $S$). The external work due to applied force is given by

$$W = \int_0^t \left( \int_S \mathbf{t} \cdot \mathbf{v} \, dS \right) \, dt + \int_0^t \left( \int_V \mathbf{b} \cdot \mathbf{v} \, dV \right) \, dt$$

where $\mathbf{v}$ is the velocity field vector, $\mathbf{t}$ the exterior surface traction vector and $\mathbf{b}$ the body force vector.

The cohesive surface contribution, representing the crack and the process zone in front of the crack tip, is described by the integral over the internal surface $S$. In this formulation, $\sigma_n$ denotes the cohesive strength, i.e. the maximum traction value that can be sustained within the cohesive zone. The cohesive length $\delta$ is the value of the displacement jump across the crack surfaces at which the stress carrying capacity of the cohesive elements reaches its maximum value. By creating new surfaces, the traction and the stiffness of the cohesive zone elements connecting these newly created surfaces are made to vanish, but the displacements across them are still continuous. During finite element analysis, the amount of external work, elastic work, plastic work and other dissipative work is calculated. Energy balance given by Eq.(3) is maintained in all FE computations.

2.3. Finite element implementation

The cohesive surface contribution as shown in Figure 3 is implemented into an FE code for eight node elements based on cohesive zone concepts. After making convergence studies, the shaft is discretized with 4108 plane strain 8 node quadrilateral elements. The length of the elements $\ell$, where the length of shaft $20\ell$. Near the crack tip, the size of the element is $1/20\ell$. A total of 4833 nodes are used to model the geometry. The fracture process zone is assumed to be a straight-line, and this straight-line is modeled by 8 node rectangular cohesive elements having zero thickness. 38 cohesive elements are used. One face of the
cohesive elements is connected to continuum elements with 4 nodes, while the other face is given symmetric displacement boundary conditions. Thus, an artificial interface is created along the fracture process zone.

![Diagram of cohesive elements connected to continuum elements with artificial interface along the fracture process zone.]

Figure 3. Representation of the fracture process using CZM in FE.

### 2.4. Definition of cohesive properties

The maximum cohesive traction is assumed to be the same as the yield strength of material of shaft, namely $\sigma_n = 250$ MPa. The maximum separation at the end of the elastic zone is $\delta_1 = 10 \, \mu$m. The interface stiffness or penalty stiffness $K_p$ can be obtained directly from $K_p = \sigma_n / \delta_1$ and is used to ensure a stiff connection between the surfaces of the material discontinuity. The values of the penalty stiffness should be selected such that the connection between the two elements and numerical stability are guaranteed.

### 3. BREATHING CRACK SIMULATION

Generally the vibration response of the cracked rotor is small, so that the bending moment due to dynamic forces (external forces and inertia forces) is smaller than the static bending moments due to external forces (such as the weight and any other stationary force, in horizontal heavy rotors of industrial plants [19]). Therefore, the breathing behaviour is dominated by the static bending moment and the dependence of the stiffness variation on the vibration response can be neglected. The breathing mechanism of a Jeffcott rotor with cracked shaft on rigid supports has been investigated. The principle of this simulation is summarized in Figure 4.

Several assumptions are made: the location of the crack is assumed to be known at the mid-span of the shaft which is the same as the maximum deflection point. Only a single transverse crack has been considered in the shaft. The plane strain condition is assumed at the crack front due to the geometry constraint. Both ends of the shaft are rigidly supported. The shaft has a uniform circular cross-section. Length $L$ and diameter $d$ of the shaft are assumed 1.0 m and 0.08 m respectively. The material of the shaft is considered to be homogeneous.
Figure 4. Principle of the breathing crack simulation using FE and MBS.

and isotropic. Young’s modulus $E$, Poisson’s ratio $\nu$ and mass density $\rho$ are 210 GPa, 0.3 and 7850 kg/m$^3$, respectively. Yield strength $\sigma_Y$ and ultimate strength of material $\sigma_{max}$ are 250 MPa and 400 MPa, respectively.

4. DYNAMIC BEHAVIOUR OF ROTATING FLEXIBLE CRACKED SHAFT

The FE model of the elastic cracked shaft is transferred into a MBS in order to compute the displacements due to the dynamic loads acting during rotation at different rotating speeds. Although the duration of each step of the breathing mechanism is relatively long and time consuming and the breathing mechanism results are based on transverse vibration response of a rotating shaft from MBS software, by reducing the number of degrees of freedom of the FE model, it has been shown that breathing mechanism in rotating shafts can be reproduced by FE and MBS. The aim of the study is to analyse the additional deflection due to the breathing crack during one revolution of the shaft. Using MBS, the inertia forces can be observed and taken into account and the transverse vibration response results are used as deflection input for the FE model in order to predict the breathing mechanism during rotation of shaft.

The breathing mechanism generated by the rotating bending load used in the literature has some limitations. Due to the presence of inertia forces, the dynamic behaviour of rotating structures is different from those of static structures. Although in case of weight dominance, the amplitude of the vibration response due to inertia forces is smaller than due to weight forces, elastic forces and presence of a crack (if the shaft is assumed to be balanced), using the inertia force into account will yield more accurate results. The 3D finite element calculations allow the breathing mechanism to be predicted accurately. The breathing mechanism is strongly influenced by the weight of the shaft. In many publications on the breathing crack,
assumptions on the breathing mechanism have been made. For transverse cracks in rotating shafts, the variation of the crack front line perpendicular to the crack front has been extensively addressed by Darpe et al. [20]. This breathing crack form is also used by Jun et al. [21], and Sinou and Lees [22]. An elliptical shape has been proposed by Bachschmid et al. [19], and Shih [23]. Based on finite element model simulation and some reported experimental results and in order to model more realistically the breathing crack mechanism during rotation, the breathing crack shape is modelled by a parabolic shape, that opens and closes due to bending stresses (Liong and Proppe [24]).

The idea is that the vibration responses in the centroid of the shaft obtained from MBS software are exported into FE model in order to analyse the breathing mechanism, as schematically shown in Figure 5. The opening crack is simulated for one cycle of revolution of the cracked shaft specimen in steady state condition. The breathing mechanism is generated by the bending due to external load (weight) by increasing the angle by steps of $\pi/12$ rad. The breathing (open and closed crack areas are evaluated in each angular step) is observed by the nodal displacement and the stress distribution (tensile or compressive stress) around the crack. The prediction of the breathing mechanism was performed by the following steps: for each angle of rotation, stress distribution due to the bending moment is recorded over the cross section. Compressive (negative) stresses indicate the closed region. The crack opens where zero or very small positive numerical values of stresses appear and the contact forces vanish. Displacements and stresses can be observed at the crack surfaces.

Results of the relative crack depth versus the shaft rotation angle during breathing are shown in Figure 6. Four functions are used to approximate the relative crack depth, namely linear, quadratic, harmonic and power function. Table 1 gives the approximations for a relative crack depth $a/d = 0.1$ as function of shaft rotation angle $\theta$. It is shown that harmonic and quadratic function have minimal error and the nearest correlation with the simulation results. In this sense, the relative crack depth during crack opening should be understood not as linear increasing but as harmonic function or quadratic polynomial.
Figure 6. Curve fitting of breathing of the simulation results.

Table 1. Curve Fitting of the simulation results for \( a/d \) = 0.1, \( \theta \) in [rad]

<table>
<thead>
<tr>
<th>Model</th>
<th>Mathematical model</th>
<th>Sum of square error</th>
<th>R-square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear function</td>
<td>( a/d = 0.039\theta + 0.001 )</td>
<td>0.000 580</td>
<td>0.951 6</td>
</tr>
<tr>
<td>Quadratic polynomial</td>
<td>( a/d = -0.012\theta^2 + 0.076\theta - 0.020 )</td>
<td>0.000 011</td>
<td>0.999 0</td>
</tr>
<tr>
<td>Harmonic function</td>
<td>( a/d = 0.010 \sin(0.593\theta - 0.145) )</td>
<td>0.000 006</td>
<td>0.999 5</td>
</tr>
<tr>
<td>Power function</td>
<td>( a/d = 0.123\theta^{0.373} - 0.077 )</td>
<td>0.000 099</td>
<td>0.991 7</td>
</tr>
</tbody>
</table>

Due to the presence of gravity, the upper portion of the cracked rotor at the beginning of a rotation is under compression and the crack is closed. As the rotor continues to rotate and the gravity direction being constant, the upper part now comes in the lower tensile region causing the crack to open. The accuracy of the results is demonstrated through comparison with results available in the literature. As long as the relative crack depth is small (\( a/d \leq 0.2 \)), the model of breathing crack parallel to crack front line or straight line may be used [24]. In comparison with the crack closure straight line model, the simulation results are only different for rotation angles less than \( \pi/2 \). Figure 7 displays the comparison between simulation results and crack closure straight line model for a shallow crack \( a/d = 0.1 \), from the beginning until the crack opens completely. With respect to the crack closure straight line model, it can be observed that the relative crack depth \( a/d \) during the rotation is different from the simulation. Crack opening of the crack closure straight line model increases with the same amount for every step of angle of rotation, i.e. \( a(\theta) = a \theta/\pi \) where \( a \) is crack depth and \( \theta \) is angle of rotation until half revolution (\( \pi \) radian). It can be observed that crack opening of the straight line model increases faster than crack opening of the simulation until an angle of \( \pi/3 \).

Another model to describe the breathing crack is a bilinear model as shown in Figure 8. The crack begins opening at shaft rotation angle \( \theta_0 \) and at crack front angle \( \alpha \) that defines the angle between two crack front lines. The shaft rotates with shaft rotation angle \( \theta \) and the
Figure 7. Comparison between simulation results and crack closure straight line model for a shallow crack \( a/d = 0.1 \).

Crack front angle \( \alpha \) increases until maximum crack width \( b_{\max} \) is reached. The crack front angle \( \alpha \) and the crack width \( b \) non-linearly increase during rotation from \( \theta_0 \) to \( \theta_{\max} \) (Figure 9 and 10). One can obtain the crack front angle \( \alpha \) and the crack width as function of the shaft rotation angle respectively by

\[
\alpha = \begin{cases} 
0 & \text{crack still closed} \\
-0.018\theta^3 + 0.016\theta^2 + 0.35\theta + 2.4 & \theta \leq \theta_0 = \frac{\pi}{6} \\
\theta_0 \leq \theta \leq \theta_{\max} & \text{crack opens} \\
\theta \geq \theta_{\max} = \frac{5\pi}{6} & \text{crack opens completely}
\end{cases}
\]

(6)
\[
\alpha = -0.084\theta^2 + 0.51\theta + 2.4 \\
\alpha = -0.018\theta^3 + 0.016\theta^2 + 0.35\theta + 2.4
\]

Figure 9. Crack front angle \(\alpha\) during rotation of shaft.

\[
\frac{b}{b_{\max}} = \begin{cases} 
0 & \theta \leq \theta_0 = \frac{\pi}{6} \\
0.076\theta^4 - 0.54\theta^3 + 1.1\theta^2 - 0.23\theta - 0.0056 & \theta_0 \leq \theta \leq \theta_{\max} = \frac{5\pi}{6} \\
1 & \theta \geq \theta_{\max} = \frac{5\pi}{6}
\end{cases}
\]

Figure 10. Normalized crack width \(\frac{b}{b_{\max}}\) during rotation of shaft.

For the relative crack depth \(a/d = 0.1\), the crack begins to open at \(\theta_0 = \pi/6\) and \(\alpha = 5\pi/6\) while the crack opens completely at \(\theta_{\max} = 5\pi/6\) and \(\alpha = \pi\). Therefore, the crack front angle and the crack width during crack opening are

\[
\alpha = -0.018\theta^3 + 0.016\theta^2 + 0.35\theta + 2.4 \\
\frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}
\]

\[\frac{b}{b_{\max}} = 0.076\theta^4 - 0.54\theta^3 + 1.1\theta^2 - 0.23\theta - 0.0056 \quad \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6}\]

In order to get more accurate interpretation, the relative crack area (i.e., ratio of the partially open crack area to the fully open crack area) with respect to the angle of rotation is
used as shown in Figure 11. Results are similar when the crack closes. The relative crack area of simulation results is in good agreement with the crack closure straight line model between rotation angles $\pi/3$ and $5\pi/6$ and with the bilinear model in the range from 0 and $\pi$, but there are some relevant differences with respect to the crack closure perpendicular line model used by Darpe et al. [20]. Another interpretation, the area moment of inertia about the rotation axis can be approximated. Finally, the stiffness of the cracked shaft can be evaluated (stiffness is found to vary linearly with the area moment of inertia). The normalised stiffness (ratio between stiffness of the cracked shaft and stiffness of the uncracked shaft) is presented as shown in Figure 12. It is shown that the normalised stiffness between simulation results and the crack closure bilinear model is very close, which validates the bilinear model.

Figure 11. Comparison of the relative crack area between simulation results, crack closure straight line, perpendicular line and bilinear model for a shallow crack $a/d=0.1$.

Figure 12. Comparison of the normalised stiffness between simulation results, crack closure straight line, perpendicular line and bilinear model for a shallow crack $a/d=0.1$. 
5. CONCLUSIONS

In this study, a simulation process of FE and MBS is employed. A cracked elastic shaft is modelled by FE which is transferred into an MBS model in order to study the stiffness variation due to breathing at different rotating speeds. The vibration responses in the centroid of the shaft obtained from MBS have been transferred again into an FE model in order to observe the breathing mechanism. The results have shown that the breathing mechanism is influenced by the vibration due to inertia forces, by rotating speed and by relative crack depth. It is shown that the relative crack depth during crack opening should be understood not as linear increasing but as harmonic function or quadratic polynomial. It can be noted that as long as the relative crack depth is small, the model of breathing crack parallel to crack front line (crack closure straight line model) or the bilinear model may be used. The main difference with respect to the crack closure straight line is that the crack opening in the simulation is not constant at the beginning. In order to get more accurate interpretation, the relative crack area (i.e. ratio of the partially open crack area to the fully open crack area) with respect to the angle of rotation is used. The simulated relative crack area is in good agreement with the crack closure straight line model between rotation angles $\pi/3$ and $5\pi/6$ and with the bilinear model in the range from $0$ and $\pi$. Furthermore, the area moment of inertia about the rotation axis can be approximated. From this information, the stiffness variation of the shaft can be evaluated and a breathing steering function can be calibrated. This function may be used either in analytical or one dimensional finite element formulations in order to evaluate changes in the natural frequencies or to carry out stability analysis of the cracked rotor. The normalised stiffness (ratio between stiffness of the cracked shaft and stiffness of the uncracked shaft) has shown that the normalised stiffness between simulation results and the crack closure bilinear model is very close, which validates the bilinear model accurately.

In light of the presented results and the conclusions, several subjects can be recommended for future research. First of all, the CZM could be extended to consider plasticity and crack propagation of the cracked shaft. Another important point on which the knowledge could be improved is the prediction of crack propagation on cracked rotor and residual life estimation from static loads and from the dynamical behaviour of cracked rotors. The CZM can be easily implemented in FEM to analyze the dynamic behaviour of a cracked shaft, is recommended to use CZM to study different types of cracks such as longitudinal and slant cracks. Some other parameters such as internal damping, unbalance and thermal transients could be studied to obtain results for their effect on the breathing mechanism as well as on stability of vibration. Further analysis on crack morphology is extremely important to understand the dynamic behaviour of cracked shaft. This would include shallow and wide cracks. Effects of rotating speed on breathing mechanism could also be of interest.
6. REFERENCES


