Recycling for sequences of linear systems by proper orthogonal decomposition augmented Krylov subspaces

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Abstract

Sequences of sparse linear systems \( A_i x_i = b_i, i = 1, \ldots, n \) arise in many applications, including iterative methods for PDE-constrained optimization. In such cases, inexact solves are often sufficient to guarantee convergence, wherein a forcing sequence \( \{ \eta^i \}_{i=1}^n \) defines modest solver tolerances \( \eta^i > 0 \) such that the computed solutions \( \tilde{x}_i \) satisfy \( \| A_i \tilde{x}_i - b_i \|_2 \leq \eta^i \). Krylov-subspace recycling methods accelerate convergence for such problems by reusing (truncated) information generated during the solution of previous linear systems. In particular, they search for solutions in the subspace \( K_m + Y \), where the subspace \( Y \) contains information from previous solves and ‘augments’ the Krylov subspace \( K_m \). Existing recycling techniques (e.g., deflation, optimal truncation) are tailored to improve convergence toward exact solutions; as a result, they do not always realize convergence acceleration when inexact solutions are sought. Instead, we propose a recycling technique inspired by model reduction that aims to efficiently compute inexact solutions. In particular, we construct \( Y \) by a goal-oriented proper orthogonal decomposition (POD) of previous search directions, where the POD inner product enables efficient computation of solutions in \( Y \).

Keywords: Krylov-subspace recycling, model reduction, sequence of linear systems, truncation